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A THERMAL ANALYSIS OF THE M140 COUNTERRECOIL SPRING

TECHNICAL REPORT

Dr. William J. Leech

November 1971

RESEARCH DIRECTORATE

WEAPONS LABORATORY AT ROCK ISLAND

RESEARCH, DEVELOPMENT AND ENGINEERING DIRECTORATE

U. S. ARMY WEAPONS COMMAND

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A thermal analysis of the M140 counterrecoil spring was carried out by the Research Directorate, Weapons Laboratory at Rock Island. An investigation was made (1) to determine the significant physical parameters influencing temperature rise of the counterrecoil spring when this spring rubs against the wall of the cradle recoil-mechanism, and (2) to obtain data for design changes to reduce the temperature rise. A mathematical model of the frictional heating process was developed and programmed for numerical evaluation. A parametric study was made to determine the dominant physical parameters. The dominant heat sink was determined to be the wall of the cradle recoil mechanism. The maximum temperature rise was found to be a strong function of the contact area between the spring and the wall. Heat losses to the hydraulic oil were insignificant. The analysis indicates that the M140 Gun Mount could be improved in design by a coating of a thin layer of material (having high thermal conductivity) being applied to the interior wall of the cradle recoil mechanism, and by a preflattening of the exterior of the first few coils of the counterrecoil spring. (U) (Leech, William J.)

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ABSTRACT

A thermal analysis of the M140 counterrecoil spring was carried out by the Research Directorate, Weapons Laboratory at Rock Island. An investigation was made (1) to determine the significant physical parameters influencing temperature rise of the counterrecoil spring when this spring rubs against the wall of the cradle recoil-mechanism, and (2) to obtain data for design changes to reduce the temperature rise. A mathematical model of the frictional heating process was developed and programmed for numerical evaluation. A parametric study was made to determine the dominant physical parameters. The dominant heat sink was determined to be the wall of the cradle recoil mechanism. The maximum temperature rise was found to be a strong function of the contact area between the spring and the wall. Heat losses to the hydraulic oil were insignificant. The analysis indicates that the M140 Gun Mount could be improved in design by a coating of a thin layer of material (having high thermal conductivity) being applied to the interior wall of the cradle recoil mechanism, and by a preflattening of the exterior of the first few coils of the counterrecoil spring.

CONTENTS

INTRODUCTION

The counterrecoil spring in the M140 gun mount has had a high rate of failure because of the formation of cracks on the outer diameter of the spring. The results of previous investigations[1,2,3] indicate that the failures probably occur because the hardness of the spring material is increased above design limits owing to frictional heating and subsequent quenching in the hydraulic oil. This investigation was undertaken to determine which parameters affect the frictional heating when the recoil spring rubs against the recoil mechanism cradle. The relative effect of each parameter on the heating process has been determined.

Erickson and Rhee[4] have investigated the temperature changes caused by frictional heating when an insulated semi-infinite block rubs against a semi-infinite plane. The plane was found to be the dominant heat sink. Expressions were derived with which the interface temperatures are related to the thermal properties, velocity of the rubbing plane, action of the tangential force on the interface, and the area of the rubbing surface. They obtained the following expression for the mean steady-state interface temperature:

$$\bar{T}_s = \frac{4}{3\sqrt{\pi}} \mu F_n \left(\frac{V}{K\rho CL}\right)^{\frac{1}{2}} \tag{1}$$

In the analysis given in this report, a section of the recoil spring that rubs against the cradle wall is considered. The effects of convective cooling of the spring by hydraulic oil are included. Time-dependent piston velocity is included in the analysis. Solutions for various combinations of parameters were obtained by numerical evaluation.

A program was initiated to obtain data for experimental verification of the analytic solution. However, due to a withdrawal of funding the experimental portion of the investigation had to be terminated before meaningful results had been obtained.

The results of the analytic investigation show which parameters have the most significant effect on spring heating and thus indicate which design changes would most effectively reduce spring temperatures.

THEORETICAL ANALYSIS

The simplified physical model that was isolated for detailed analysis is shown in Figure 1. In this figure, a section of spring coil in contact with the cradle wall is illustrated. A normal force per unit length, F_n, acts on the spring, and a section of the spring surface, of width L, is in contact with the wall. A relative velocity, V, and a constant friction coefficient, μ, exist between the spring and the wall. Hydraulic oil surrounds the spring and is in contact with the wall.

Heat is generated when a relative velocity exists between the two surfaces in contact. The rate of heat generation is proportional to the product of the tangential force acting on the interface and the velocity. The tangential force is equal to the product of the normal force and the friction coefficient. The generated heat can be absorbed by the spring, the cradle wall, and the hydraulic oil. The heat is transferred to the spring and wall by conduction, and to the hydraulic oil by convection from both the spring and the wall. As the temperature of the interface increases, the wall becomes the dominant heat sink. This occurs because the wall provides a much larger mass to absorb the heat. The overall temperature rise in the wall is not so great as that in the spring, and the interface temperature gradients in the wall become much greater than those in the spring. The existence of this phenomenon may be made more apparent by the realization that the physical situation illustrated in Figure 1 is analagous to a cool fluid (the wall) flowing over a hot surface (the spring) with heat generation at the surface. The majority of the heat would be absorbed by the cooler fluid.

The assumption is that axial conduction in the spring coil may be disregarded. Additional assumptions are that perfect thermal contact exists between the spring and the wall, that all thermal properties are constant, and that the hydraulic oil remains at a constant temperature. The origin of the coordinate system for the wall is attached to the spring.

The governing differential equations are deduced from the general energy equations in the Appendix. The governing differential equations for the wall and the spring are

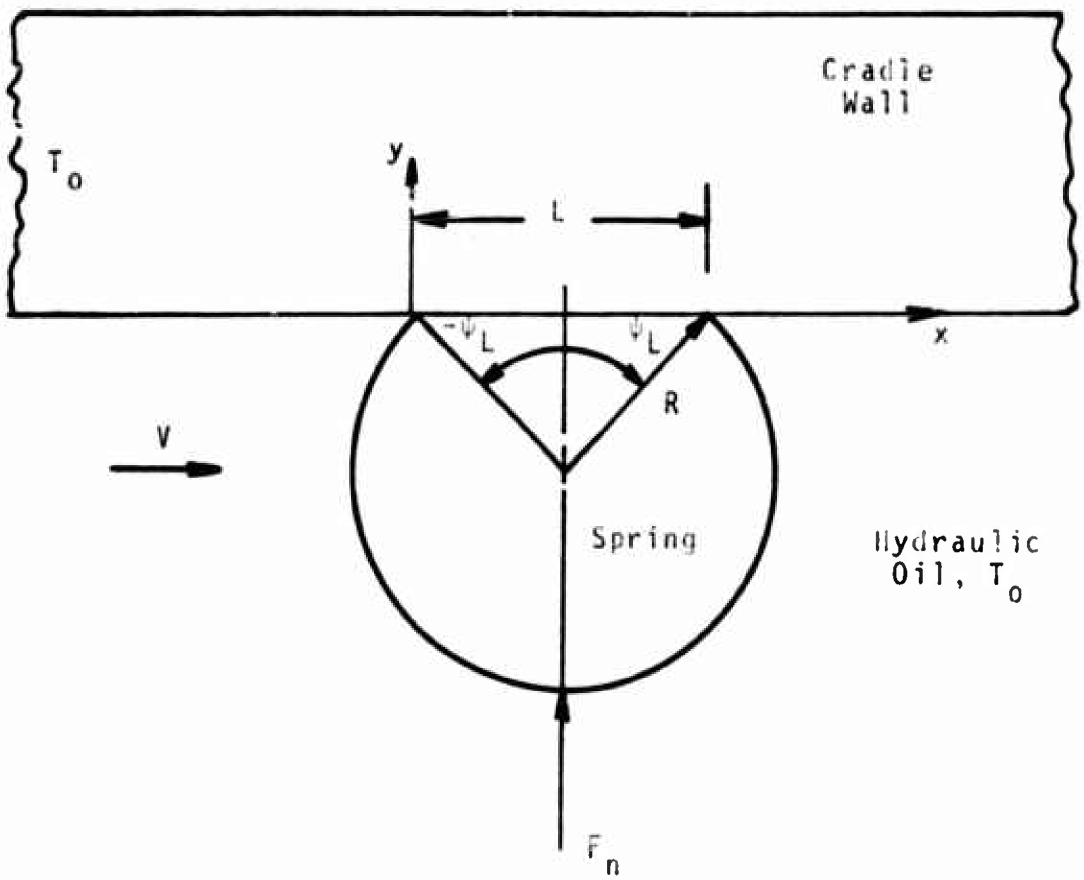

FIGURE 1 SIMPLIFIED PHYSICAL MODEL

given by

$$\frac{\partial T_w}{\partial x} = \frac{\alpha_w}{V} \frac{\partial^2 T_w}{\partial y^2} \tag{2}$$

and

$$\frac{\partial T_s}{\partial t} = \alpha_s \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \psi^2}\right] \tag{3}$$

The boundary conditions for the wall are

$$-K_w \frac{\partial T_w}{\partial y}(x,o) = q_w, \quad 0<x<L \tag{4}$$

$$T_w(x,\infty) = T_o \tag{5}$$

$$T(o,y) = T_o \tag{6}$$

The boundary conditions for the spring are given by

$$-K_s \frac{\partial T_s}{\partial r}(R_1, \psi, t) = q_s''(t), \quad -\psi_L<\psi<\psi_L \tag{7}$$

$$-K_s \frac{\partial T_s}{\partial r}(R_1, \psi, t) = h_c[T_s(R,\psi,t)-T_o], \quad \psi_L<\psi<-\psi_L \tag{8}$$

and the initial conditions are

$$T_s(r,\psi,0) = T_o \tag{9}$$

In Equation 2, one-dimensional quasi-steady heat conduction in the wall is described. The one-dimensional quasi-steady analysis is shown in the Appendix to be valid in this case.

Equation 4 is obtained under the assumption that the heat flux applied to the wall is constant and uniform over the interface. Temporal variations in wall heat flux have no significant effect if quasi-steady conditions prevail. A variation exists in wall heat flux along the interface, even though the heat generation by friction is uniform. This variation occurs because the interfacial temperature gradients in the y direction are greater for small values of x than for larger values. The assumption of uniform heat flux is still valid provided temperature gradients in the y direction are much greater than in the x direction over most of the interface. This condition is satisfied as shown in the Appendix. In Equation 7, the heat flux to the spring is given as uniform over a circumferential section of the outer radius. The heat flux is actually applied over a flat section of spring. The circumferential section closely approximates the flat section, provided the angular interval is not large. Since $L<R\Delta\psi$, the approximation leads to a conservative estimate of interfacial temperature rise because, in the actual case, the heat is being applied over a slightly smaller area.

The solution of Equation 2, subject to the specified boundary conditions, is[5]

$$T_w(x,y)-T_o = \frac{2q_w''}{K_w}\left\{\left(\frac{\alpha_w x}{\pi V}\right)^{1/2}\exp\frac{-Y^2V}{4\alpha_w x} - \frac{Y}{2}\,\mathrm{erfc}\left[\frac{Y}{2}\left(\frac{V}{\alpha_w x}\right)^{1/2}\right]\right\} \tag{10}$$

where erfc(z) is defined as

$$\mathrm{erfc}(z) = 1-\mathrm{erf}(z) = 1 - \frac{2}{\sqrt{\pi}}\int_0^z e^{-n^2}\,dn \tag{11}$$

The interface temperature (y=0) is determined from Equation 10 as

$$T_w(x,o)-T_o = \frac{2q_w''}{K_w}\left[\frac{\alpha_w x}{\pi V}\right]^{1/2} \tag{12}$$

The mean interface temperature is determined by the integration of Equation 12 to yield

$$\bar{T} - T_o = \frac{4}{3} \frac{q_w''}{K_w} \left(\frac{\alpha_w L}{\pi V}\right)^{\frac{1}{2}} \tag{13}$$

The interface temperature reaches a maximum at x=L and is

$$T_L - T_o = \frac{2q_w''}{K_w} \left(\frac{\alpha_w L}{\pi V}\right)^{\frac{1}{2}} \tag{14}$$

thus,

$$T_L - T_o = \frac{3}{2} (\bar{T}-T_o) \tag{15}$$

Equation 15 indicates that the maximum temperature rise is one and one-half times greater than the mean temperature rise.

Equation 13 may be rearranged to give

$$q_w'' = \frac{3K_w}{4} \left(\frac{\pi V}{\alpha_w L}\right)^{\frac{1}{2}} (\bar{T}-T_o) \tag{16}$$

Equation 16 gives the heat flux to the wall in terms of the unknown mean interface temperature.

The total heat flux at the interface is due to frictional heat generation and is given by

$$q_t'' = \left|\frac{\mu F_n V}{JL}\right| \tag{17}$$

The portion of the total heat flux applied to the spring is

$$q_s'' = q_t'' - q_w'' \tag{18}$$

or

$$q_s'' = \left|\frac{\mu F_n V}{JL}\right| - \frac{3K_w}{4}\left(\frac{\pi V}{\alpha_w L}\right)^{\frac{1}{2}} (\bar{T}-T_0) \tag{19}$$

According to Equation 15, the wall heat flux increases in proportion to the mean interface temperature. A corresponding decrease occurs in the heat flux to the spring as the interface temperature increases. The interface tends to some steady-state maximum value if the velocity remains constant. Equation 19 may be substituted into Equation 7 to give the boundary condition for the spring in terms of interface temperatures.

The solution, for the temperature distribution in the spring, was obtained by numerical evaluation. The results of the numerical evaluations are given in the following section.

ANALYTICAL RESULTS

The temperature distributions in the M140 spring, during recoil, were determined numerically. Thermal properties, dimensions, and time-velocity data used were the actual values for the M140 recoil spring. Variable parameters included the width of the rubbing surface, normal force acting on the spring, friction coefficient, and convective heat-transfer coefficient between the spring and the hydraulic fluid.

The M140 recoil springs are currently being fabricated from 9262H steel, which has a silicon content of 2 per cent. The use of maraging steel, having a nickel content from 17 to 19 per cent, for replacement of the current material has been investigated.[3] Therefore, results were obtained for spring materials consisting of steel having a silicon content of 2 per cent and also of steel having a nickel content of 18 per cent to determine the effect on the temperature distribution due to a material change.

Eckert and Drake[6] give the following property values used in the calculations:

	2 Per Cent Silicon Steel	18 Per Cent Nickel Steel
$\rho_s(\frac{lb_m}{ft^3})$	479	499
$Cp_s(\frac{BTU}{lb\ ^\circ F})$	0.11	0.11
$K_s(\frac{BTU}{hr\ ft ^\circ F})$	18	12
$\alpha_s(\frac{ft^2}{hr})$	0.344	0.204

The outside radius of the spring stock is 0.5175 inch. Velocity-time data for the recoil piston were obtained from K. W. Maier's report[7], and are reproduced in Figure 2. The following values of the variable parameters were used in the numerical evaluation:

$$L = 0.05,\ 0.1,\ 0.2,\ 0.3 \text{ inches} \tag{20}$$

$$\mu F_n = 10,\ 50,\ 100,\ 200\ lb_f/in \tag{21}$$

$$h_c = 0,\ \infty\ \ BTU/hr\ ft^2\ ^\circ F \tag{22}$$

$$K_w = 18,\ 36\ \ BTU/hr\ ft\ ^\circ F \tag{23}$$

$$\alpha_w = 0.344,\ 0.688\ \ ft^2/hr \tag{24}$$

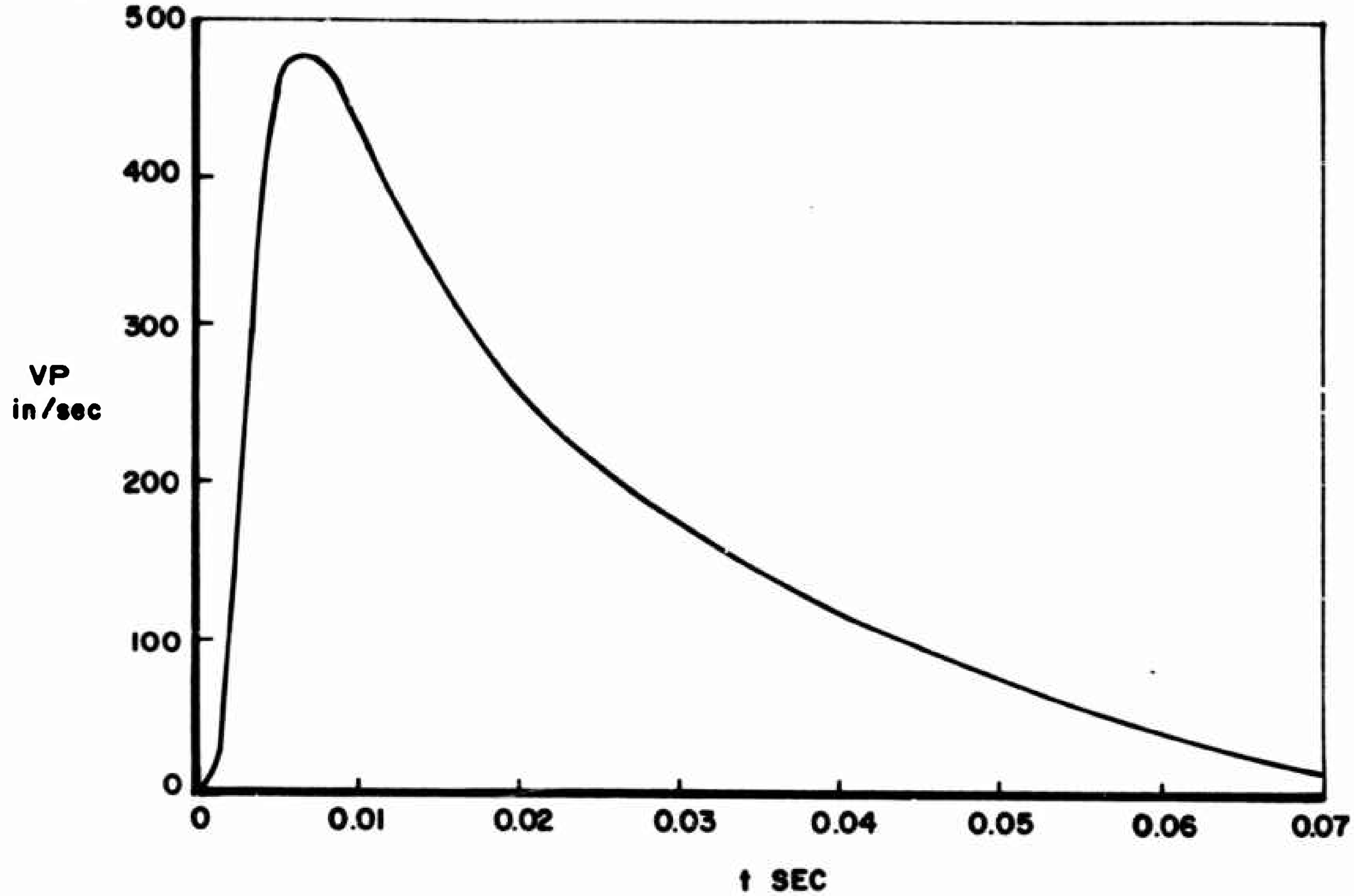

FIGURE 2. VELOCITY-TIME DATA FOR THE M-140 COUNTER RECOIL PISTON

Results were obtained for combinations of these parameters, and these results are given in graphical form in Figures 3, 4, 5, 6, and 7. Plots of the maximum interface temperature during recoil versus interface width as functions of the tangential force, F_n, and of thermal conductivities of the spring and wall are shown in Figure 3. The width of the rubbing surface has a significant effect on the interface temperature. For small values of L, the maximum interface temperature becomes large and approaches infinity as the width approaches zero. These conditions occur because the heat is being applied over a small area. As the width of the thickness is increased, the curves become flat, and a point is reached at which a further increase in rubbing width has no appreciable effect on the reduction of the interface temperature. The dominance of the wall on the interfacial temperature is also apparent. Doubling the thermal conductivity of the wall results in a large reduction of interface temperature, whereas a large change in the conductivity of the spring has no significant effect. The same data are plotted in Figure 4 with log-log coordinates. The data lie on straight lines; this indicates a relationship of the form

$$T_{max} \sim L^m \tag{25}$$

The values of m, obtained from the slope of the curves, are -0.48 for tangential forces of 50, 100 and 200 lb_f, but -0.26 for a tangential force of 10 lb_f. Thus, the value of m for this geometry is not constant, but is dependent upon the tangential force.

Erickson and Rhee found that m=-0.5, independent of tangential force, when considering a semi-infinite block rubbing against a semi-infinite plane with a constant velocity.

The data points are plotted as T_{max} versus μF_n in Figure 5. This results in a family of straight lines, passing through the origin, which indicates a relation between T_{max} and μF_n of the form

$$T_{max} \sim \mu F_n \tag{26}$$

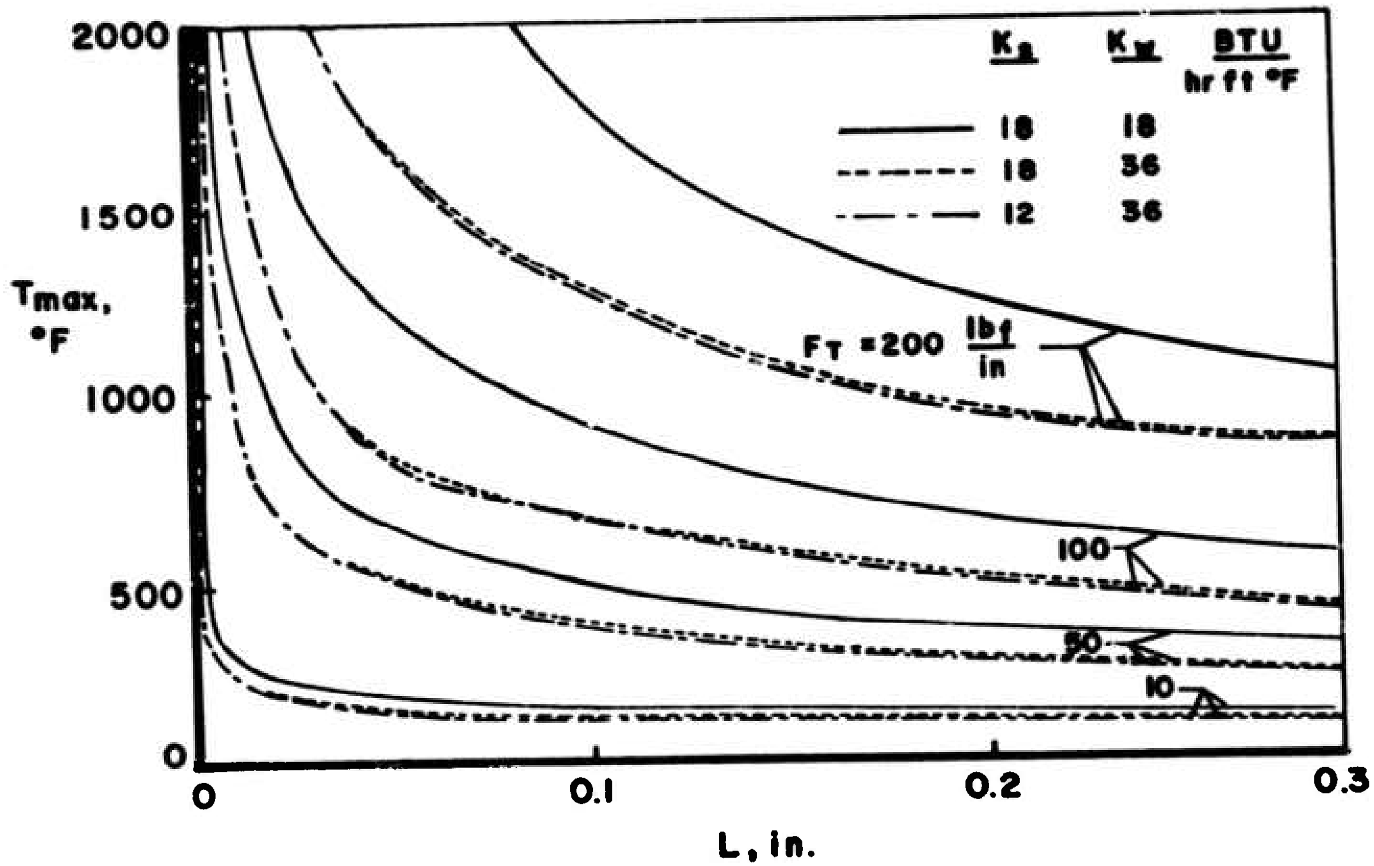

FIGURE 3. MAXIMUM INTERFACE TEMPERATURE VERSUS RUBBING WIDTH AS A FUNCTION OF TANGENTIAL FORCE AND THERMAL CONDUCTIVITY.

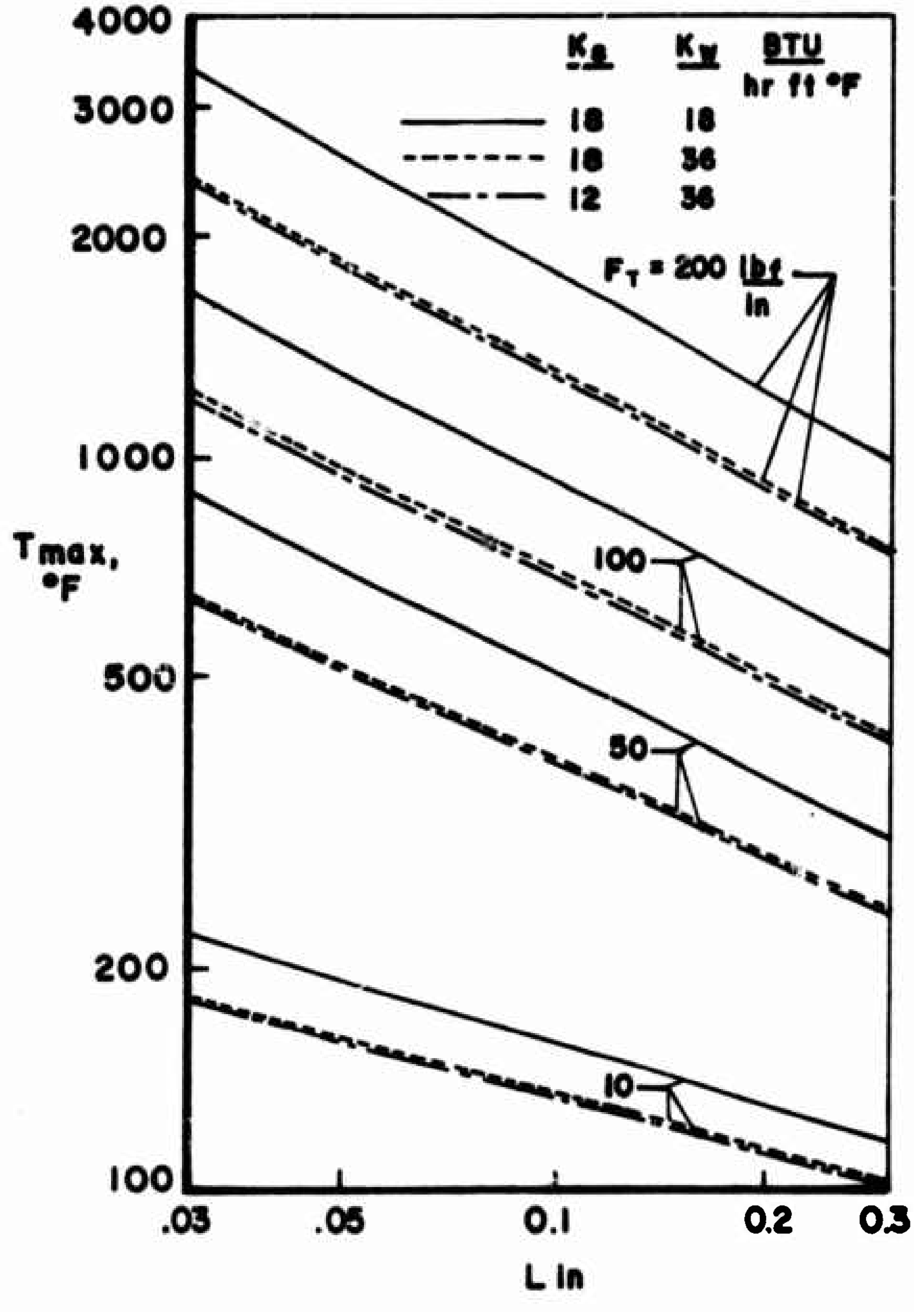

FIGURE 4. LOG - LOG PLOT OF MAXIMUM INTERFACE TEMPERATURE VERSUS RUBBING WIDTH AS A FUNCTION OF TANGENTIAL FORCE AND THERMAL CONDUCTIVITY.

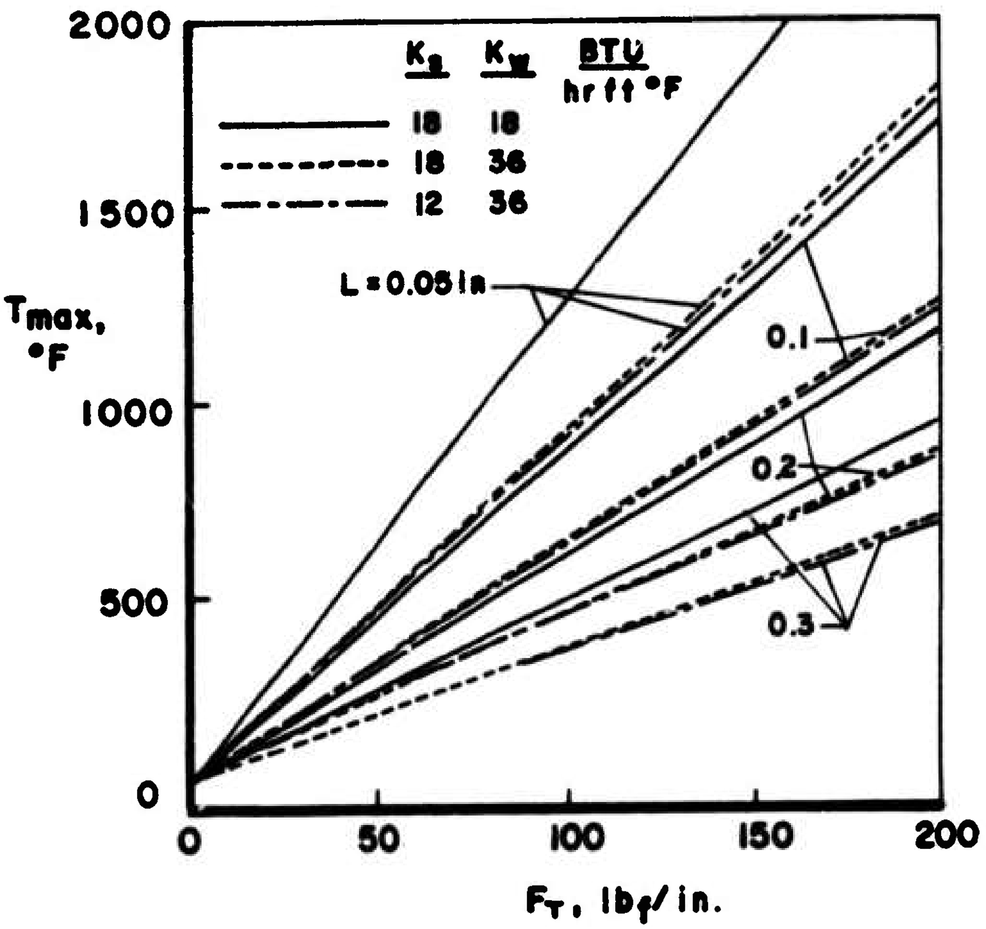

FIGURE 5. MAXIMUM INTERFACE TEMPERATURE VERSUS TANGENTIAL FORCE AS A FUNCTION OF RUBBING WIDTH AND THERMAL CONDUCTIVITY.

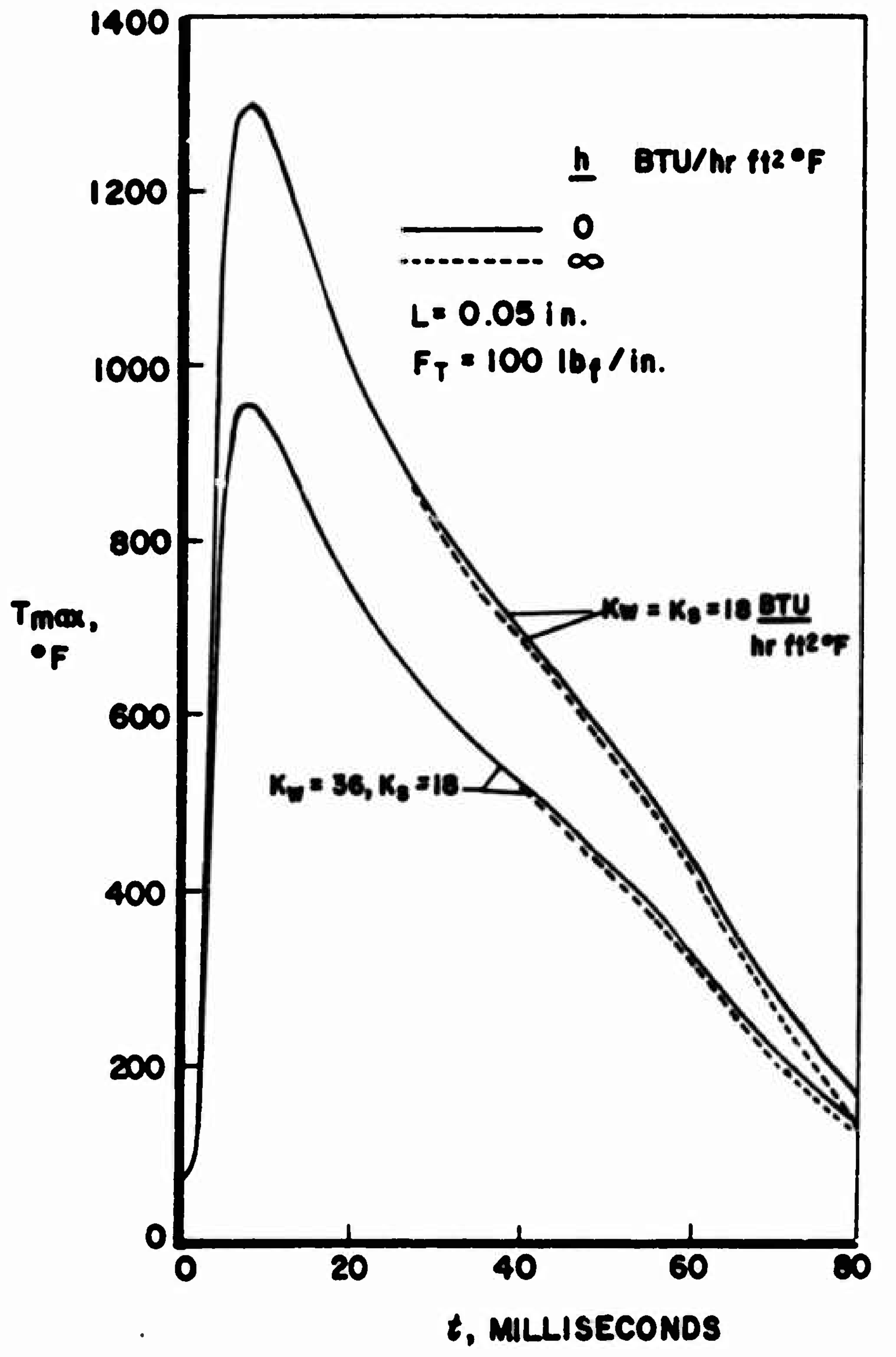

FIGURE 6. MAXIMUM TEMPERATURE VERSUS TIME AS A FUNCTION OF CONVECTION COEFFICIENT AND THERMAL CONDUCTIVITY.

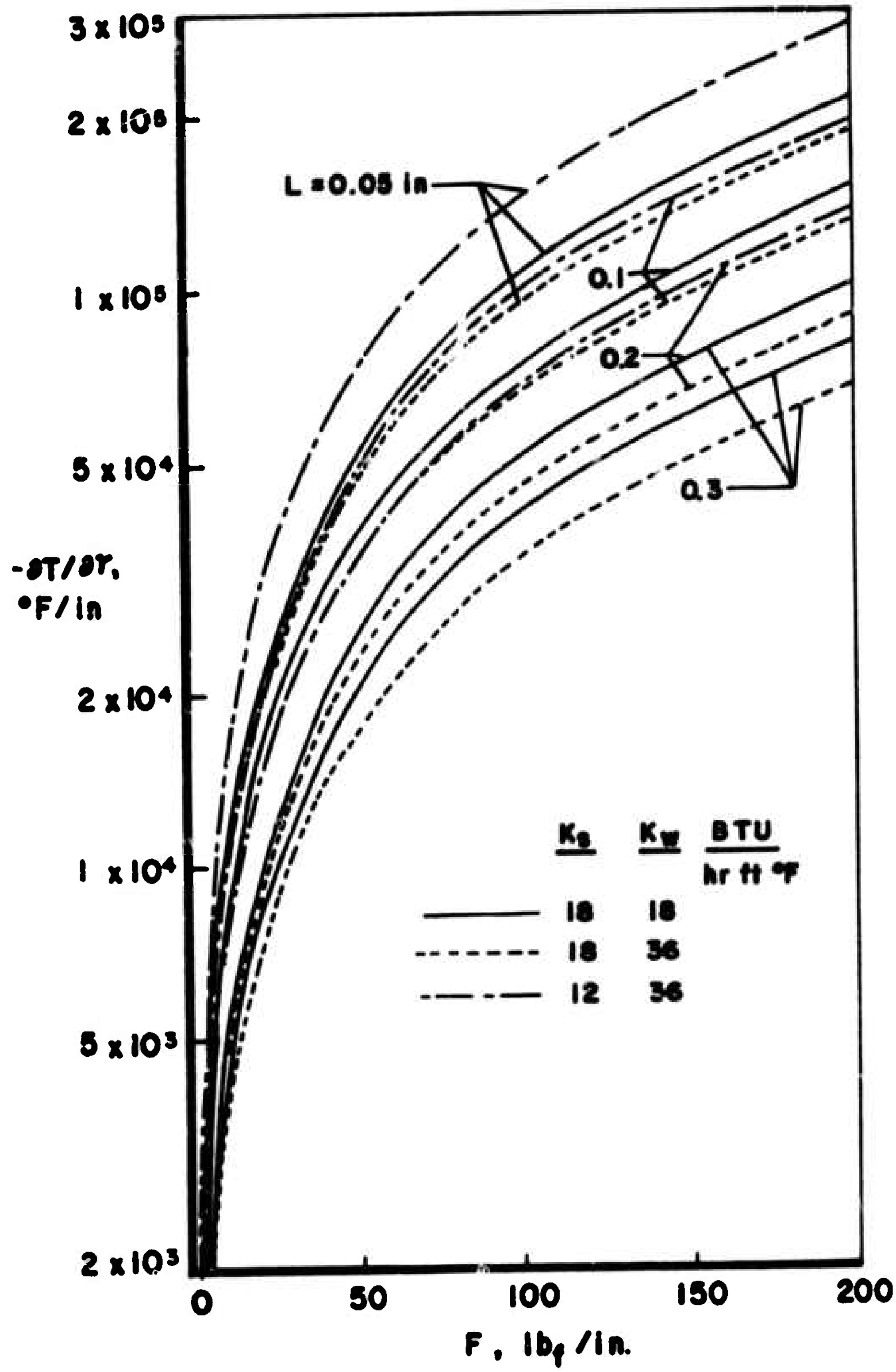

FIGURE 7. MAXIMUM TEMPERATURE GRADIENT AT THE SPRING SURFACE VERSUS TANGENTIAL FORCE AS A FUNCTION OF RUBBING WIDTH AND THERMAL CONDUCTIVITY.

This is the same form obtained by Erickson and Rhee. A plot of T_L versus time for several values of wall conductivity and for heat-transfer coefficients of zero and infinity are shown in Figure 6. Heat losses to the oil obviously have an insignificant effect on the interface temperature. This occurs because the temperature rise is localized in the region near the interface. The temperature differences between the spring and the hydraulic oil are small over most of the spring surface. An increase in the convective cooling capacity of the hydraulic oil offers no prospect of a decrease in the interfacial temperatures. Again, the dominance of the wall as a heat sink is indicated in Figure 6. The maximum temperature is attained during a short time span and it corresponds closely with the time of maximum piston velocity. This shows that the temperature rise occurs rapidly with a small time lag. Thus, large temperature increases could occur even if the spring were in contact with the wall for a short time, if the normal forces and velocities during contact were large.

A plot of the maximum temperature gradient in the spring at the interface is shown in Figure 7. Note that an increase in the thermal diffusivity of the wall decreases the temperature gradient, but a decrease in the diffusivity of the spring increases the temperature gradient. The use of maraging steel as opposed to 9262H steel has no effect on the temperature, but a 50 per cent increase in the thermal gradient at the interface occurs. Therefore, maraging steel may be less desirable than 9262H steel, if the thermal stresses at the surface are a factor in overall spring performance. This factor should be weighed against some of the other advantages of the use of maraging steel.

The analytical results show that the only way to reduce the interface temperatures, for a given tangential force and rubbing width, is to increase the thermal conductivity of the recoil mechanism wall. This can be done by the coating of the wall with a thin layer of highly conductive material. A thin layer is sufficient because the thermal penetration distance in the wall is small and the coating has only to be as thick as the penetration distance. The thermal penetration distance is given by

$$\delta \sim \frac{\alpha_w}{V} \tag{27}$$

At the maximum value of piston velocity, the penetration distance for most metals is in the order of 10^{-5} inches, so a very thin coating is sufficient. Williams[8] has shown that the coating of the recoil mechanism wall with molybdenum reduces spring wear. Molybdenum has a high thermal conductivity and is also desirable from a thermal point of view.

EXPERIMENTAL PROGRAM

An experimental program was initiated to check the validity of the mathematical model. An experiment was designed and run, which consisted of the rubbing of a steel block against a steel plate. The steel block was mounted in the jig of a planing machine, and the plate was attached to the planer table. Both, the normal force and the table speed, could be varied. Thermocouples were mounted on the block at the interface to monitor the temperature. Strain gages were attached to the block to measure the tangential forces. However, only a small temperature rise could be produced because of the chattering of the block that caused the steel plane to ge gouged whenever the tangential force or the table velocity became large enough to produce a significant temperature rise. The experimental apparatus was redesigned by substitution of a stiffer block and heat-treated steel plane. The new design was submitted for fabrication, but a reduction in funding caused a cancellation of the fabrication. Therefore, the experimental program had to be terminated.

CONCLUSIONS AND RECOMMENDATIONS

Frictional heating of the M140 counterrecoil spring was analytically investigated. A mathematical model of the process was developed and programmed for numerical evaluation. A parametric study was performed to determine the effect of each parameter on the heating process. On the basis of the results of the parametric study, the following conclusions were reached:

1. The wall of the recoil mechanism cradle is the dominant heat sink.

2. The maximum temperature rise is a strong function of the rubbing width; however, an increase in the rubbing width beyond a certain value has a limited effect on temperature reduction.

3. Heat losses to the hydraulic oil have an insignificant effect on the maximum interface temperature.

4. The maximum temperature is reached in a short time interval. Thus, rubbing will cause a large temperature rise even if it occurs for a short time, during which the piston is moving at a high velocity.

5. The use of maraging steel as a spring material has no effect on the spring temperature. However, use of maraging steel will result in a 50 per cent greater thermal gradient at the interface.

The two most obvious ways to reduce or to eliminate the spring heating problem are those in which the normal forces acting on the spring are reduced and the friction coefficient between the spring and wall is reduced. However, even if accomplishment of either of these objectives is impossible, reduction of temperature rise due to frictional heating is possible. On the basis of the analytical results of this study, the following two recommendations are given for the improvement of the design of the M140 counterrecoil mechanism.

1. The thermal conductivity of the recoil mechanism wall should be increased. Coating the wall with a thin layer of material having high thermal conductivity will result in this increase. A molybdenum coating has the desirable thermal properties and has also been shown to reduce spring wear.

2. The exterior surface of the first few coils of the spring should be preflattened. Providing a rubbing width of about 0.30 inch will significantly reduce the maximum temperature rise.

LIST OF SYMBOLS

Cp_s	specific heat of spring
Cp_w	specific heat of wall
F_n	normal force per unit depth
F_T	tangential force = μF_n
h_c	heat transfer coefficient
J	Joules constant = 778 ft.lb/BTU
K_s	thermal conductivity of spring
K_w	thermal conductivity of wall
L	width of rubbing surface
ℓ	reference length
m	value of exponent
q_s''	heat flux to spring
q_t''	total heat flux
q_w''	heat flux to wall
R	outer radius of spring
r	radial coordinate
T_L	temperature of interface at $x=L$
T_o	initial temperature
T_s	temperature of spring
T_w	temperature of wall

LIST OF SYMBOLS

T_{max}	maximum temperature reached during recoil
$\bar{T}$	mean interface temperature
t	time
t_c	time constant
V	velocity
$\bar{V}$	dimensionless velocity
x	spatial coordinate
$\bar{x}$	dimensionless spatial coordinate
y	spatial coordinate
$\bar{y}$	dimensionless spatial coordinate
α_s	thermal diffusivity of spring
α_w	thermal diffusivity of wall
δ	reference length
θ_w	dimensionless wall temperature
μ	coefficient of friction
ρ_s	density of spring material
ρ_w	density of wall material
τ	dimensionless time
ψ	angular coordinate
$\Delta\psi$	contact angle

APPENDIX

GOVERNING EQUATIONS

The energy equation for the wall, illustrated in Figure 1, is

$$\frac{\partial T_w}{\partial t} + V \frac{\partial T_w}{\partial x} = \alpha_w \left[\frac{\partial^2 T_w}{\partial x^2} + \frac{\partial^2 T_w}{\partial y^2}\right] \tag{28}$$

Equation 28 is now recast in a dimensionless form by the introduction of the nondimensional variables. A reference length, ℓ, is chosen so that the value of the dimensionless temperature gradient in the x direction does not exceed unity in the region of interest, $o<x<L$. Also, the reference length in the y direction, δ, is assumed to be much smaller than the unspecified length ℓ. A time constant, t_c is chosen so that the time rate of change of dimensionless temperature has a maximum value of unity. The following nondimensional variables are defined.

$$\theta_w = \frac{T_w}{T_s} \tag{29}$$

$$\bar{x} = \frac{x}{\ell} \tag{30}$$

$$\bar{y} = \frac{y}{\delta} \tag{31}$$

$$\tau = \frac{t}{t_c} \tag{32}$$

$$\bar{V} = \frac{V}{V} = 1 \tag{33}$$

The energy equation in dimensionless form becomes

$$\frac{\partial \theta_w}{\partial \bar{x}} = \frac{\alpha_w}{V\ell} \left[\frac{\partial^2 \theta_w}{\partial \bar{x}^2} + \left(\frac{\ell}{\delta}\right)^2 \frac{\partial \theta^2_w}{\partial \bar{y}^2} - \frac{\ell^2}{\alpha_w t_c} \frac{\partial \theta_w}{\partial_w \tau_c}\right] \tag{34}$$

$$1 \qquad \delta^2 \qquad 1 \qquad \frac{1}{\delta^2} \qquad 1 \qquad\qquad 1$$

The quantity $\alpha_w/V\ell$ is assumed to be very small, i.e., V is very large. The order of magnitude of each term is shown below Equation 34. Note that conduction in the x direction is small in comparison to conduction in the y direction if

$$\left(\frac{\delta}{\ell}\right)^2 << 1 \tag{35}$$

In addition, conductive energy transport in the y direction is of the same order of magnitude of convective energy transport in the x direction, only if

$$\left(\frac{\delta}{\ell}\right)^2 \sim \frac{\alpha_w}{V\ell} << 1 \tag{36}$$

or

$$\ell >> \frac{\alpha_w}{V} \tag{37}$$

Transient effects are small in comparison to convective effects when

$$\frac{\alpha_w}{V\ell} \cdot \frac{\ell^2}{\alpha_w t_c} = \frac{\ell}{V t_c} << 1 \tag{38}$$

or when

$$t_c >> \frac{\ell}{V} \tag{39}$$

Since $\alpha_w/V\ell$ is assumed to be very small, transient effects may also be disregarded when

$$\frac{\ell^2}{\alpha_w t_c} \sim 1 \tag{40}$$

Combination of Equations 39 and 40 gives the following criterion for neglecting transient effects in the plane.

$$\frac{V\ell}{\alpha_w} >> 1 \tag{41}$$

In the analysis given above, the boundary conditions were not considered. The heat flux to the wall is a time-dependent value even if the piston velocity is constant because it is a function of the time-dependent interface temperature. Erickson and Rhee have shown that, for a constant piston velocity, transient effects due to variable interface temperature may be disregarded when

$$\frac{Vt_c}{L} \geq 32.5 \tag{42}$$

Combination of Equations 39 and 42 gives

$$\frac{V\ell}{\alpha_w} \geq 32.5 >> 1 \tag{43}$$

where $\ell \leq L$ has been substituted for L. When the piston velocity is a variable, the heat flux to the wall will also be time-dependent. The heat generation due to friction is directly proportional to the piston velocity, and the fractional variation in generated heat may be expressed as

$$\frac{\Delta q}{q} = \frac{\Delta V}{V} = \frac{dV}{dt} \cdot \frac{\Delta t}{V} \tag{44}$$

The maximum possible time interval during which heat may be applied to any point on the wall is L/V that is assumed to be larger than ℓ/V. Substitution of this value for Δt and introduction of the dimensionless variables into Equation 44 gives the following condition for disregarding transients due to variable piston velocity.

$$\frac{\Delta q}{q} = \frac{\ell}{t_c V} \frac{d\overline{V}}{d\tau} << 1 \tag{45}$$

This condition is fulfilled provided

$$t_c >> \frac{\ell}{V} \tag{46}$$

which again requires that

$$\frac{V\ell}{\alpha_w} >> 1 \tag{47}$$

Thus, all transient effects may be disregarded when the fraction $V\ell/\alpha_w$ is much greater than one.

For a piston velocity of 20 m/sec and a value of $\alpha_w = 1\ ft^2/hr$ evaluation of Equation 37 results in

$$\ell >> 0.002 \text{ in} \tag{48}$$

Thus, if the width of the rubbing surface, L, is much greater than 0.002 in., say L=0.02 in., conduction in the x direction may be disregarded. For a piston velocity of 100 in/sec and a rubbing width of 0.002 in., Equation 41 has the following value

$$\frac{VL}{\alpha} = 50 >> 1 \tag{49}$$

Thus, transient effects may also be disregarded. Therefore, the conclusion is that, for this investigation, the energy equation for the wall may be written as

$$\frac{\partial T_w}{\partial x} = \frac{\alpha_w}{V} \frac{\partial^2 T_w}{\partial y^2} \tag{50}$$

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